AVIRAL
JEE | NEET | FOUNDATIONS

WEEKLY TEST MEDICAL PLUS -03 TEST - 07 RAJPUR SOLUTION Date 01-09-2019

[PHYSICS]

1. $H = \frac{v^2 \sin^2 \theta}{2g}$ and $R = \frac{v^2 \sin^2 2\theta}{g}$

Since, R = 2H, so $\frac{v^2 \sin 2\theta}{g} = 2 \times \frac{v^2 \sin^2 \theta}{2g}$

or 2sinθ cosθ = sin²θ or or tanθ = 2

$\therefore$ $R = v^2 \times \frac{2}{g} \times \sin\theta\cos\theta$

$= \frac{2v^2}{g} \times \frac{2}{\sqrt{5}} \times \frac{1}{\sqrt{5}} = \frac{4v^2}{5g}$

2. For the preson to be able to catch the ball, the horizontal component of velocity of the ball should be same as the speed of the person, i.e.,

$v_0 \cos\theta = \frac{v_0}{2}$

or $\cos\theta = \frac{1}{2}$ or $\theta = 60^o$

3. Let, u_x = 3 m/s, a_x = 0
u_y = 0, a_y = 1 m/sec² and t = 4 sec
If v_x and v_y be the velocities after 4 sec respectively, then
$v_x = u_x + a_x t = 3\ ms^{-1}$
and $v_y = u_y + a_y t = 0 + 1 \times 4 = 4\ ms^{-1}$

$\therefore$ $v = \sqrt{v_x^2 + v_y^2} = \sqrt{3^2 + 4^2} = \sqrt{9+16} = 5\ m/s$

Angle made by the result velocity w.r.t. direction of initial velocities, i.e., x-axis, is

$\beta = \tan^{-1}\frac{v_y}{v_x} = \tan^{1}\left(\frac{4}{3}\right)$

4. $h = \frac{u^2 \sin^2 \theta}{2g}$, hence, $\frac{\Delta h}{h} = 2.\frac{\Delta u}{u}$

Since, $\frac{\Delta u}{u} = 2\%$, hence, $\frac{\Delta h}{h} = 4\%$

5. $R_{max.} = R = \frac{u^2}{g}$ or $u^2 = Rg$

Now, as $\text{range} = \frac{u^2 \sin 2\theta}{g}$

then, $\frac{R}{2} = \frac{Rg \sin 2\theta}{g}$

or $\sin 2\theta = \frac{1}{2} = \sin 30^\circ$

or $\theta = 15^\circ$

6. The horizontal range is the same for the angles of projection θ and $(90^\circ - \theta)$

$$t_1 = \frac{2u \sin\theta}{g}$$

$$t_2 = \frac{2u \sin(90^\circ - \theta)}{g} = \frac{2u \cos\theta}{g}$$

$$\therefore \quad t_1 t_2 = \frac{2u \sin\theta}{g} \times \frac{2u \cos\theta}{g} = \frac{2}{g}\left[\frac{u^2 \sin 2\theta}{g}\right] = \frac{2}{g} R$$

where $R = \frac{u^2 \sin 2\theta}{g}$

Hence, $t_1 t_1 \propto R$.

7. Range = 150 = ut and

$$h = \frac{15}{100} = \frac{1}{2} \times gt^2$$

or $t^2 = \frac{2 \times 15}{100 \times g} = \frac{30}{1000}$

or $t = \frac{\sqrt{3}}{10}$

$$\therefore \quad u = \frac{150}{t} = \frac{150 \times 10}{\sqrt{3}} = 500\sqrt{3} \text{ ms}^{-1}$$

8. $y = 12x - \frac{3}{4}x^2$

$$\frac{dy}{dt} = 12\frac{dx}{dt} - \frac{3}{2} x \frac{dx}{dt}$$

At x = 0: $\frac{dy}{dt} = 12\frac{dx}{dt}$

If θ be the angle of projection, then

$$\frac{dy/dt}{dx/dt} = 12 = \tan\theta$$

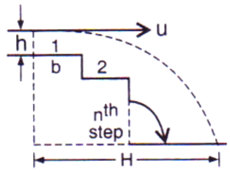
Also, if u = initial velocity, then $u\cos\theta = 3$

Hence, $\tan\theta \times u\cos\theta = 36$ or $u \sin\theta = 36$

Range, $R = \frac{u^2 \sin 2\theta}{g} = \frac{2u^2 \sin\theta \cos\theta}{g}$

$$= \frac{2(u\sin\theta)(u\cos\theta)}{10} = \frac{2 \times 36 \times 3}{10} = 21.6 \text{ m}$$

9. If the ball hits the nth step, the horizontal distance traversed = nb.
Vertical distance traversed = nh.
Here, velocity along horizontal direction = u.
Velocity along vertical direction = 0.



∴ nb = ut .....(i)

$nh = 0 + \frac{1}{2}gt^2$ ....(ii)

From eqn. (i),

$t = \frac{nb}{u}$, ∴ $nh = \frac{1}{2}g \times \left(\frac{nb}{u}\right)^2$

∴ $n = \frac{2hu^2}{gb^2}$

10. Potential energies at the highest point are equal to the loss in kinetic energies. That is $\frac{1}{2}mu^2$ and $\frac{1}{2}m(u\cos 60^o)^2$ or $\frac{1}{4} \times \frac{1}{2}mu^2$.

11. $t = \frac{2u\sin\theta}{g} = \frac{2 \times 20 \times \sin 30^o}{10} = 2\ s$

Now, we shall calculate the total time taken by the ball to hit the ground.

Using, $s = ut + \frac{1}{2}gt^2$,

we get; $40 = -10t' + \frac{1}{2} \times 10(t')^2$

[∵ u = – 20 sin30º = – 10 m/s]

∴ 5(t′) – 10t′ – 40 = 0
Solving, we have, t′ = 4 s

∴ $\frac{t'}{t} = \frac{4s}{2s} = \frac{2}{1}$

12. $H_{max.} = \frac{u^2\sin^2\theta}{2g}$

$T = \frac{2u\sin\theta}{g}$

$\frac{H_{max.}}{T^2} = \frac{u^2\sin^2\theta}{2g} \times \frac{g^2}{4u^2\sin^2\theta} = \frac{g}{8} = \frac{10}{8} = \frac{5}{4}$

13.

14. $H = \frac{u^2\sin^2\theta}{2g}$ or $80 = \frac{u^2\sin^2\theta}{2 \times 10}$

or $u^2\sin\theta = 1600$
or $u\sin\theta = 40\ ms^{-1}$.
Horizontal velocity = u cosθ = at
= 3 × 30 = 90 ms^{-1}

$$\frac{u\sin\theta}{u\cos\theta} = \frac{40}{90}$$

or $\tan\theta = \frac{4}{9}$ or $\theta = \tan^{-1}\left(\frac{4}{9}\right)$

15. $h = (u\sin\theta)t - \frac{1}{2}gt^2$

$d = (u\cos\theta)t$

or $t = \frac{d}{u\cos\theta}$

$\therefore$ $h = u\sin\theta.\frac{d}{u\cos\theta} - \frac{1}{2}g.\frac{d^2}{u^2\cos^2\theta}$

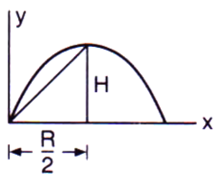
$\therefore$ $u = \frac{d}{\cos\theta}\sqrt{\frac{g}{2(d\tan\theta - h)}}$

16. Average velocity $= \frac{\text{Displacement}}{\text{Time}}$

$$v_{av.} = \frac{\sqrt{H^2 + \frac{R^2}{4}}}{T/2} \qquad \text{......(i)}$$

Here, H = maximum height

$= \frac{v^2\sin^2\theta}{2g}$



$R = \text{range} = \frac{v^2\sin 2\theta}{g}$

and $T = \text{time of flight} = \frac{2v\sin\theta}{g}$

$\therefore$ $v_{av.} = \frac{v}{2}\sqrt{1 + 3\cos^2\theta}$

17. $T = \frac{2u_y}{g}$, $H = \frac{u_y^2}{2g}$

and $R = u_x T$

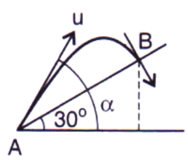
When a horizontal acceleration is also given to the projectile u_y and T and H will remain unchanged while the range will become

$R' = u_x T + \frac{1}{2}aT^2$

$= R + \frac{1}{2}\frac{g}{4}\left(\frac{4u_y^2}{g^2}\right) = R + H$

18. t_{AB} = time of flight of projectile $= \frac{2u\sin(\alpha - 30^o)}{g\cos 30^o}$

Now component of velocity along the plane becomes zero at point B.



$\therefore$ $0 = u\cos(\alpha - 30^o) - g\sin 30^o \times T$

or $u\cos(\alpha - 30^o)$

$= g\sin 30^o \times \frac{2u\sin(\alpha - 30^o)}{g\cos 30^o}$

or $\tan(\alpha - 30^o) = \frac{\cot 30^o}{2} = \frac{\sqrt{3}}{2}$

$\therefore \alpha = 30^o + \tan^{-1}\left(\frac{\sqrt{3}}{2}\right)$

19. Horizontal component of velocity,

$u_H = u\cos 60^o = \frac{u}{2}$

$\therefore AC = u_H \times t = \frac{ut}{2}$

and AB = AC sec 30º

$= \left(\frac{ut}{2}\right)\left(\frac{2}{\sqrt{3}}\right) = ut/\sqrt{3}$

20. For the projectile to pass through (30 m, 40 m)

$= 40 = 30\tan\alpha - \frac{g(30)^2}{2u^2}(1+\tan^2\alpha)$

or $900\tan^2\alpha - 6u^2\tan\alpha + (900 + 8u^2) = 0$

For real value of α,

$(6u^2)^2 \geq 3600\,(900 + 8u^2)$

or $(u^4 - 800u^2) \geq 900{,}00$

or $(u^2 - 400)^2 \geq 25{,}000$

or $u^2 \geq 900$

or $u \geq 30$ m/s.

21. Let P be the position of projectile when it is moving with velocity u at an angle to the horizon and Q the position when the direction of path makes an angle β with horizontal.

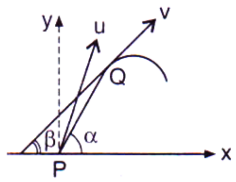
Take P as origin and horizontal and vertical lines through it as axes.

Suppose time from P to Q is t, then v is the velocity at Q.

hence $v = \cos\beta = u\cos\alpha$ ....(i)

$v\sin\beta = u\sin\alpha - \sin\alpha - gt$ .....(ii)

Dividing equation (ii) by equation (i)



$\tan\beta = \frac{u\sin\alpha - gt}{u\cos\alpha}$

$t = \frac{u\cos\alpha(\tan\alpha - \tan\beta)}{g}$ ....(iii)

If in the position Q, the angle turned through be θ, then $\alpha - \beta = \theta$

or $\beta = \alpha - \theta$

From eqn. (iii), $t = \frac{u\cos\alpha}{g}[\tan ga - \tan(\alpha - \theta)]$

$= \frac{u\cos\alpha}{g}\left[\frac{\sin\alpha}{\cos\alpha} - \frac{\sin(\alpha-\theta)}{\cos(\alpha-\theta)}\right]$

$= \frac{u\cos\alpha}{g}\frac{\sin\theta}{\cos\alpha\cos(\alpha-\theta)}$

$= \frac{u\sin\theta}{g\cos(\theta-\alpha)}$

22. $100 = (V_t - V_m)10$, $V_t = 15$ m/s

In second case : $100 = (V_t + V_m)t$

From 1st equation : $V_m = 5$ m/s

From 2nd equation : t = 5 sec

23.

24. Time taken by body A, $t_1 = 5$ sec
Acceleration of body A = a_1
Time taken by body B, $t_2 = 5 - 2 = 3$ sec
Acceleration of body B = a_2
Distance covered by first body in 5th second after start,

$$s_5 = u + \frac{a_1}{2}(2t_1 - 1)$$

$$= 0 + \frac{a_1}{2}(2 \times 5 - 1) = \frac{9a_1}{2}$$

Distance covered by the second body in the 3rd second after is start,

$$s_3 = u + \frac{a_2}{2}(2t_2 - 1)$$

$$= 0 + \frac{a_2}{2}(2 \times 3 - 1) = \frac{5a_2}{2}$$

Since, $s_5 = s_3$

$\therefore \quad \frac{9a_1}{2} = \frac{5a_2}{2}$ or $a_1 : a_2 = 5 : 9$

25. P $\xrightarrow{v_p}$ Q $\xrightarrow{v_Q}$ $v_P - v_Q = 2.4$ m/s

P $\xrightarrow{v_P}$ $\xleftarrow{v_Q}$ Q $v_P + v_Q = 6.0$ m/s

$\therefore \quad v_P = 4.2$ m/s; $v_Q = 1.8$ m/s

26. For the motion of first ball,
u = 0, a = g, t = 3s.
Let S_1 be the distance covered by the first ball in 3 sec.

$$\therefore \quad S_1 = ut + \frac{1}{2}gt^2 = 0 + \frac{1}{2} \times 10 \times (3)^2 = 45 \text{ m}$$

Let S_2 be the distance covered by the second ball in 2 sec. Then

$$S_2 = 0 + \frac{1}{2} \times 10 \times (2)^2 = 20 \text{ m}$$

$\therefore$ Separation between the two balls
$= S_1 - S_2 = 45 - 20 = 25$ m.

27.

28. **Given** : At time t = 0, velocity, v = 0.

Acceleration, $f = f_0\left(1 - \frac{t}{T}\right)$

At f = 0, $0 = f_0\left(1 - \frac{t}{T}\right)$

Since, f_0 is a constant, $\therefore 1 - \frac{t}{T} = 0$ or $t = T$

Also, acceleration, $f = \frac{dv}{dt}$

$$\therefore \quad \int_0^{v_x} dv = \int_{t=0}^{t=T} f dt = \int_0^T f_0\left(1 - \frac{t}{T}\right)dt$$

$$\therefore \quad v_x = \left[f_0 t - \frac{f_0 t^2}{2T}\right]_0^T = f_0 T - \frac{f_0 T}{2T} = \frac{1}{2} f_0 T$$

29. $S_2 = \frac{1}{2}gt_2^2 = \frac{10}{2} \times (3)^2 = 45$ m

$S_1 = \frac{1}{2}gt_1^2 = \frac{10}{2} \times (5)^2 = 125$ m

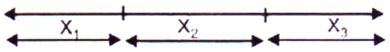
$\therefore$ S – S = 125 – 45 = 80 m

30. $S = ut + \frac{1}{2}gt^2$

$30 = -25t + \frac{10}{2}t^2$ or $t^2 - 5t - 6 = 0$

or (t – 6) (t + 1) = 0

$\therefore$ t = 6 sec

32. 

Starting from rest $x_1 = \frac{1}{2}a(10)^2$ ....(1)

$x_1 + x_2 = \frac{1}{2}a(20)^2$ ....(2)

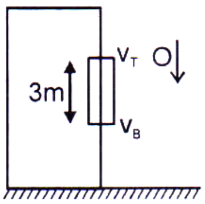
$x_1 + x_2 + x_3 = \frac{1}{2}a(30)^2$ ....(3)

From (2) – (1) $x_2 = \frac{1}{2}a(300)$

From (3) – (2) $x_3 = \frac{1}{2}a(500)$

$\Rightarrow$ $x_1 : x_2 : x_3 :: 1 : 3 : 5$

33. $s = \frac{(u+v)}{2}t$



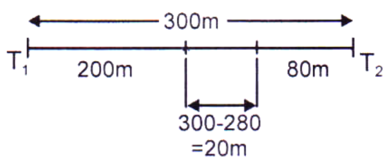
$3 = \frac{(v_T + v_B)}{2} \times 0.5$

$v_T + v_B = 12$ m/s

Also, $v_B = v_T + (9.8)(0.5)$ .... (2)

$v_B - v_T = 4.9$ m/s

34. Initial distance between trains is 300 m. Displacement of 1st train is calculated by area under V-t. curve of train 1 = $\frac{1}{2} \times 10 \times 40 = 200$ m.



Displacement of train 2 = $\frac{1}{2} \times 8 \times (-20) = -80$ m.

Which means it moves towards left.

$\therefore$ Distance between the two is 20 m.

35. Let a be the retardation produced by resistive force, t_a and t_d be the time of ascent and descent respectively. If the particle rises upto a height h

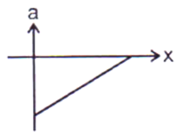
then $h = \frac{1}{2}(g + a)\,t_a^2$ and $h = \frac{1}{2}(g - a)\,t_d^2$

$\therefore \quad \frac{t_a}{t_d} = \sqrt{\frac{g-a}{g+a}} = \sqrt{\frac{10-2}{10+2}} = \sqrt{\frac{2}{3}}$ **Ans.** $\sqrt{\frac{2}{3}}$

36. The linear relationship between V and x is
$V = -mx + C$ where m and C are positive constants.
$\therefore$ Acceleration

$a = V\frac{dV}{dx} = -m(-mx + C)$ $\therefore$ $a = m^2x - mC$

Hence the graph relating a to x is :

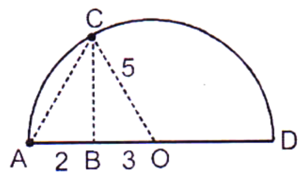


37. From triangle BCO $\Rightarrow$ BC = 4

From triangle BCA $\Rightarrow$ $AC = \sqrt{2^2 + 4^2} = 2\sqrt{5}$

$AC = u_1\,t,\quad BC = u_2\,t$

$\therefore \quad \frac{u_1}{u_2} = \frac{AC}{BC} = \frac{2\sqrt{5}}{4} = \frac{\sqrt{5}}{\sqrt{4}}$



38. After 10 sec

$\overrightarrow{u_B} = 2 \times 10 = 20$

A ⟵⟶ B

$x = 1/2 \times a \times 10^2$
$= 100$

Now $x_A = (40\,t)$

$x_B = 100 + (ut) + \frac{1}{2}(2)\,t^2 = 100 + 20\,t + t^2$

A will be ahead of B when

$x_B < x_A \quad \Rightarrow \quad 100 + 20\,t + t^2 < 40\,t$

$\Rightarrow \quad t^2 - 20\,t + 100 < 0$

$t^2 - 10t - 10\,t + 100 < 0$

$t(t - 10) - 10\,(t - 10) < 0$

$(t - 10)^2 < 0$

which is not possible

39. From given graphs : a_x is +ve & a_y is –ve, as v_x is increasing in +ve direction and v_y in –ve direction. (Checked from slope)

40. Distance travelled from time 't–1' sec to 't' sec is

$S = u + \frac{a}{2}(2t - 1)$ ....(1)

from given condition $S = t$ .... (2)

(1) & (2) $\Rightarrow t = u + \frac{a}{2}(2t - 1) \Rightarrow u = \frac{a}{2} + t(1 - a)$.

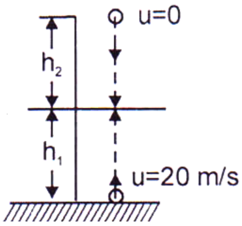
Since u and a are arbitrary constants, and they must be constant for every time.

$\Rightarrow$ coefficient of t must be equal to zero.

$\Rightarrow$ $1 - a = 0 \Rightarrow a = 1$ for $a = 1$, $u = \frac{1}{2}$ unit

Initial speed is $\frac{1}{2}$ unit. Ans.

41. Height of the building



$H = h_1 + h_2$

$= \frac{1}{2}gt^2 + ut - \frac{1}{2}gt^2$

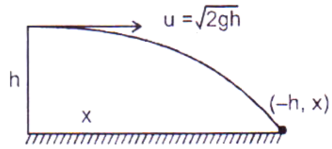
$= ut = 60$ m.

42. $\vec{r} = (t^2 - 4t + 6)\hat{i} + t^2\hat{j}$; $\vec{v} = \frac{d\vec{r}}{dt} = (2t - 4)\hat{i} + 2t\hat{j}$, $\vec{a} = \frac{d\vec{v}}{dt} = 2\hat{i} + 2\hat{j}$

if $\vec{a}$ and $\vec{v}$ are perpendicular

$\vec{a}.\vec{v} = 0$ $(2\hat{i} + 2\hat{j}).((2t - 4)\hat{i} + 2t\hat{j}) = 0$ $8t - 8 = 0$ $t = 1$ sec.

43. Using equation of trajectory :



$-h = x\tan(0°) - \frac{gx^2}{2(2gh)(\cos^2 0°)}$

$\Rightarrow x = 2h$ **Ans.**

Method II

time of flight $T = \sqrt{\frac{2h}{g}}$

horizontal distance covered during time of flight is

$x = u_x t = \sqrt{\frac{2h}{g}} \times \sqrt{2hg} = 2h$

44. Ranges for complementary angles are same

$\therefore$ Required angle $= \frac{\pi}{2} - \frac{5\pi}{36} = \frac{13\pi}{36}$ Ans.

45. Use $\alpha = \beta = 45°$ in the formula for Range down the incline plane.